Grade 7 Circumference

7.SS.1	
Demonstrate an understanding of circles by • describing the relationships among radius, diameter, and circumference of circles • relating circumference to PI • determining the sum of the central angles • constructing circles with a given radius or diameter • solving problems involving the radii, diameters, and/or circumferences of circles	1. Illustrate and explain that the diameter is twice the radius in a circle. 2. Illustrate and explain that the circumference is approximately three times the diameter in a circle. 3. Explain that, for all circles, PI is the ratio of the circumference to the diameter (C/D) , and its value is approximately 3.14. 4. Explain, using an illustration, that the sum of the central angles of a circle is 360^{o}. 5. Draw a circle with a given radius or diameter with and without a compass. 6. Solve a contextual problem involving circles.

Clarification of the outcome:

- ✦ The main part of the outcome concerns the relationship between radius/diameter and the circumference of a circle, and solving problems that involve circumference.
- ✦ A secondary part of the outcome concern central angles and their sum (360 degrees).
- ✦ The outcome also concerns students encountering a new type of number, namely PI This type of number cannot be expressed exactly as a fraction. Such numbers are referred to as irrational numbers. We use approximations for them when doing real world calculations. In the case of PI, 3.14 or 22/7 are reasonable approximations for its value.
- ✦ The term ‘circumference’ is a special word for the perimeter of a circle.
- ✦ The achievement indicator (#4) concerning central angles can be viewed as “circumference” in the sense of a radius sweeping out a circle as the radius makes a complete rotation. Other than that, the indicator is best seen as an “orphan”.

Required close-to-at-hand prior knowledge:

- ✤ Familiarity with circles, angles, and angle measurement.

SET SCENE stage

The problem task to present to students:

Provide students with the information that the orbit of the moon around the earth is approximately a circle and that the average distance from the earth to the moon is 381 500 km.

Organize students into groups of 2. Ask the groups to figure out the perimeter of the moon's orbit (how far the moon travels in one revolution around the earth).

Comments:

The main purpose of the task is to engage students in a circumference problem that serves as a core reason for learning about circumference relationships and PI.

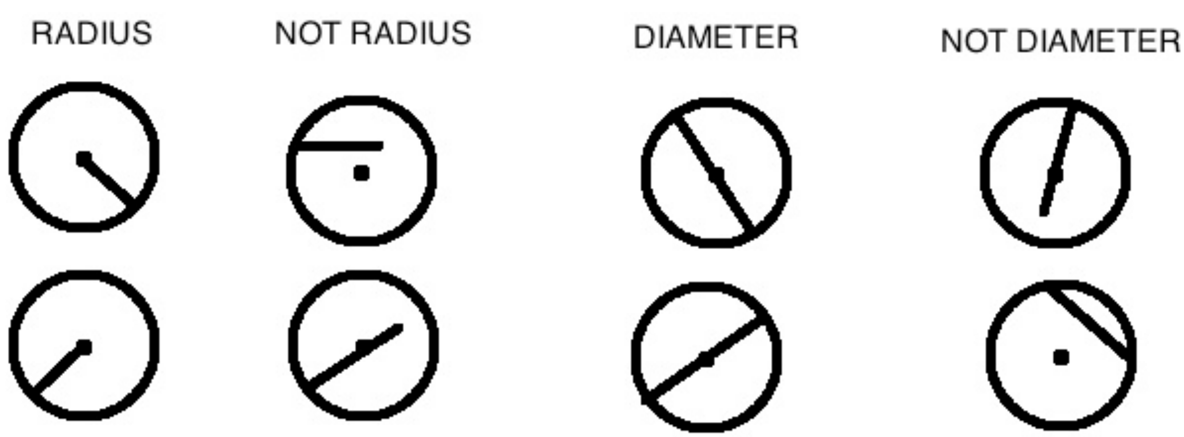
DEVELOP stage

Activity 1: Revisits SET SCENE, and addresses achievement indicators 5 and 6.

- Ask selected groups to present their approach to solving the SET SCENE task. Accept all strategies. Mention that the upcoming lesson is about a method calculating the perimeter of a circle.
- Have students brainstorm where circles and circular measurements are used in the real world. Discuss the examples.

Activity 2: Addresses achievement indicators 1 and 5.

- Tell students they first need to learn some technical language about circles. Introduce the terms radius, diameter, circumference by using an example/counter example teaching strategy. [Refer to sample diagram for radius and diameter.] Ask students to provide definitions of radius, diameter, circumference, based on the example/counter example images.



- Organize students into six teams. Each team figures out a way to do a charade for each of the three terms (radius, diameter, circumference). Each team does its charades for the three terms. The students watching try to figure out which term is being acted out each time.
- Provide two different radii. Have students draw the circles, given the radius, using a compass. Have students draw the circles, given the radius, not using a compass.
- Provide two different diameters. Have students draw the circles, given the diameter, using a compass. Provide two different diameters. Have students draw the circles, given the diameter, not using a compass.
- Ask students the relationship between radius and diameter. Ensure they realize that diameter is twice the radius and that radius is half of the diameter.

Activity 3: Addresses achievement indicators 1 and 2.

- Select an appropriate-sized can and distribute it to students. Discuss using a unit of length that is the same length as the diameter of the circle (formed by the can) to measure the circumference of the circle. This unit of length is a non-standard unit. [Prior to this, the teacher should have made strips of paper that each are one diameter in length.]
- Distribute the strips of paper to students. Have students place a string (fishing line is excellent for this because it does not stretch easily) around the can to obtain the circumference of the circle. Have students measure the length of the circumference using the strip of paper as the unit of length. They should find that the circumference of the circle is a bit longer than 3 strips of paper (in other words, the circumference is about 3 times the diameter of the circle).
- Ask students how many radii are about one circumference. Ensure they realize that about 6 radius lengths can be laid around the circumference (3 diameters is 2 x 3 = 6 radii).
- Repeat the activity with a different-sized can and use its diameter as the unit of length. Encourage students to obtain the circumference as accurately as possible. Discuss the results. Students should again find that the circumference is a bit more than 3 diameters long. Ask students to describe a relationship/formula/rule that connects circumference to diameter. Ensure they realize that one way to describe the relationship is:

circumference = roughly 3 x diameter

Activity 4: Addresses achievement indicators 3, 5, and 6.

- ✦ Provide students with three cans of different diameters. Have students trace each can onto grid paper marked in centimetres. Have students wrap masking tape around each can without overlapping the tape; then remove the tape and place it beside the corresponding can. Have students measure, in centimetres, the diameter and circumference of each can, recording the diameter and circumference in a T-table, and determine if the approximate relationship is:

  circumference = 3.14 x diameter.

- ✦ Discuss 3.14 as two decimal place accuracy for a number called PI - a number that cannot be determined exactly. Talk about PI as a measure of curvature and there is no way to accurately measure the length of a curve.

Note:

- ✤ Measuring the diameter of a traced circle is not as simple as measuring the diameter for a circle drawn using a compass set. For a traced circle, there is no dot to use as the indicator of the centre of the circle. When you use a compass set to draw a circle, the place where the compass point pressed down on the paper is the centre of the circle. One way to obtain a fairly accurate measurement of the diameter of a traced circle is to slide the ruler across the circle until you obtain a measurement that is the longest (the diameter is the longest distance across a circle).
- ✤ You could use a spreadsheet to make the T-table. The advantage of using a spreadsheet is that students can easily test their ideas about the relationship between circumference and radius. The teacher will have to explain how to write a formula in a spreadsheet but this is not a complex matter for this situation. For example, the approximate relationship when entered as a spreadsheet formula could be: '= 3.14 * cell A'.

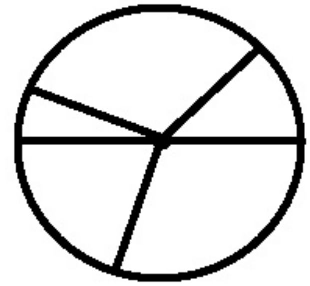
Activity 5: Addresses achievement indicators 3, 5, and 6.

- ✦ Have students estimate the circumferences of circular objects in their environment. Ask them to measure the diameters of the objects and enter the data into a spreadsheet or a T-table.
- ✦ Have students enter and use the relationship between diameter and circumference into the spreadsheet or use a calculator to calculate the circumferences. Ask them to explain why their estimates were too high/low.

Activity 6: Addresses achievement indicator 6.

- ✦ Have students solve a variety of word problems that involve radius and circumference or diameter and circumference. Two examples are: (1) *A piece of wire 4.7 m long is bent into a circle. What is the diameter of the circle?* and (2) *The tip of the minute hand of a clock travels 132 cm each hour. How long is the minute hand?*.

Activity 7: Addresses achievement indicator 4.



- ✦ Present a circle showing five central angles (see diagram). Point out the central angles. Ask students to define a central angle.
- ✦ Ask students to determine the sum of the central angles of a circle. To assist with this, present a diagram showing a circle cut into equal quarter regions and ask them to add up the four angles formed (all are right angles).
- ✦ Provide problems showing a circle with central angles, where one angle is unknown (refer to question #3 of the worksheet that follows). Ask students to solve the problems. Discuss results.

Activity 8: Revisits SET SCENE and addresses achievement indicator 6 & practice.

- ✦ Revisit the SET SCENE task about orbit of moon. Ask students to calculate the circumference of the moon's orbit using the relationship between circumference and radius/diameter.
- ✦ Ask students to research the radius of earth's orbit and Mars' orbit around the sun and research the number of days (earth days) it takes earth to orbit the sun once (365) and Mars to orbit the sun once (687).
- ✦ Ask students to calculate and compare the two orbit circumferences. Discus results.
- ✦ Ask students to calculate the speed (as km per day) of earth and the speed of Mars (as km per day) as they orbit the sun. Discus results.

Note:

Speed in km per day would be calculated by dividing the circumference of the orbit by the number of days it takes to orbit the sun.

Activity 9: Assessment of teaching.

- Provide students with a problem about circumference such as: "*One circle has a diameter of 3 metres. Another circle has a radius of 2 metres. Which circle has the greater circumference and by how much?*"

If all is well with the assessment of teaching, engage students in PRACTICE (the conclusion to the lesson plan).

An example of a partially well-designed worksheet follows.

The worksheet contains a sampling of question types. More questions of each type are needed.

The MAINTAIN stage follows the sample worksheets.

Question 1.

a) The radius of a circle is 20 cm. What is the diameter? __________

b) The diameter of a circle is 50 cm. What is the radius? __________

Question 2.

Use PI = 3

a) The radius of a circle is 12 cm. What is the circumference? __________

b) The diameter of a circle is 5 cm. What is the circumference? _________

c) The circumference of a circle is 24 cm. What is the radius? _________

d) The circumference of a circle is 30 cm. What is the diameter? _________

Question 3.

What is the size of the third central angle?

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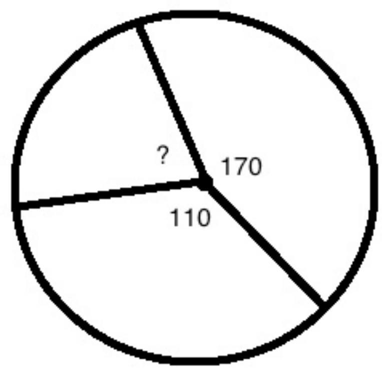


Question 4.

Solve the problem. Show your work. Use PI = 3.14

A circular tin can has a radius of 6 cm. What is the circumference of the can?

Question 5.

Solve the problem. Show your work, including a diagram. Use PI = 3.14

Sam wants to make a design of five circles, each one just touching the next one. The centres lie along the same line. The diameter of the first circle is 20 cm. The diameter of the next circle is always 5 cm longer than the diameter of the previous circle. Sam will use rope to make the circles. How much rope will Sam need?

MAINTAIN stage

Mini-task example

Every so often:

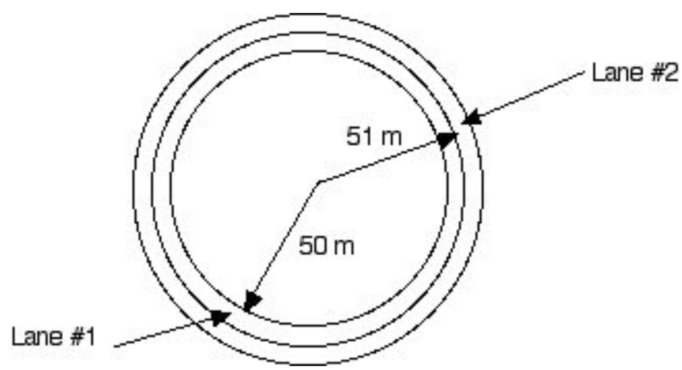
- Present a radius or diameter of a circle and ask students to calculate the circumference.

Rich-task example

Ask students to solve the following problem.

Two joggers are running around a circular running track that has 8 lanes. Runner #1 is in lane #1 (radius is 50 m) and runner #2 is in lane #2 (radius = 51 m). ***[Refer to the diagram.]***

If each jogger stays as close as possible to the inside of his/her lane, which jogger runs the longest distance for a lap and how much longer is that distance? Suppose the joggers do 20 laps. How much further does one of the joggers run?



Comments

This is a rich-task because it is a complex problem that integrates circumference with a real world situation.

* The problem indicates why lanes are staggered in Olympic track events that involve running around a circular track.